International Journal of Theoretical Physics, Vol. 24, No. 6, 1985

A Generalized Field Theory
Charged Spherical Symmetric Solution

M. I. Wanas[1,2]

Received November 30, 1984

Three solutions with spherical symmetry are obtained for the field equations of the generalized field theory established recently by Mikhail and Wanas. The solutions found are in agreement with classical known results. The solution representing a generalized field, outside a spherical symmetric charged body, is found to have an extra term compared with the Reissner–Nordström metric. The space used for application is of type FIGI, so the solutions obtained correspond to a field in a matter-free space. A brief comparison between the solutions obtained and those given by other field theories is given. Two methods have been used to get physical results: the first is the type analysis, and the second is the comparison with classical known results by writing down the metric of the associated Riemannian space.

1. INTRODUCTION

The author, in collaboration with Mikhail [Mikhail and Wanas (1977); referred to hereafter as (I)], has constructed a generalized field theory using a space admitting absolute parallelism. The same authors [Mikhail and Wanas (1981); referred to hereafter as (II)], have examined the solution of the field equations of the proposed theory in the absence of feedback effects, and the results obtained are found to be in complete agreement with corresponding classical known results. In addition, these results predict a type of interaction between gravitational and electromagnetic fields. In the same paper (II), the authors have classified, in a covariant way, spaces which can be used as models in any application of the theory to problems in physics or astronomy.

In a recent paper [Wanas (1981); referred to hereafter as (III)], the author has examined the solution of the field equations using a space of

[1]Department of Mathematics, University of London, King's College, United Kingdom.
[2]Permanent address: Department of Astronomy, Faculty of Science, Cairo University, Egypt.

the type FOGI, which represents the absence of the electromagnetic field but presence of a gravitational field which is not strong outside a spherical symmetric body. The metric of the real domain of the associated Riemannian space is found to be identical with that of the Schwarzschild exterior metric.

The aim of the present work is to explore more exact solutions using spaces of more general type. The space which we are going to use for this study is of type FIGI. This space is capable of representing a generalized field outside a spherically symmetric charged body. All notations used here are the same as those used in the previous papers (I), (II), and (III).

2. STATIONARY SPHERICAL SYMMETRIC SPACES

The structure of spaces admitting absolute parallelism with spherical symmetry has been studied by Robertson (1932). The tetrad vectors $\underset{i}{\lambda}^{\mu}$ (i, $\mu = 0, 1, 2, 3$) defining the structure of such spaces, as given by Robertson, can be written in coordinates of Cartesian type in the following form (α, β, $a \neq 0$):

$$\begin{aligned} \underset{0}{\lambda}^{0} &= A, \\ \underset{0}{\lambda}^{\alpha} &= Dx^{\alpha} \\ \underset{a}{\lambda}^{0} &= Ex^{a} \\ \underset{a}{\lambda}^{\alpha} &= Fx^{a}x^{\alpha} + \delta^{\alpha}_{a}B + S\varepsilon_{\alpha a\beta}x^{\beta} \end{aligned} \tag{1}$$

where A, B, D, E, F, S are functions of r $(=x^{\alpha}x^{\alpha})$ for the stationary case, and $\varepsilon_{\alpha\beta\gamma}$ is skew with respect to all indices where $\varepsilon_{123} = 1$. As shown by Robertson (1932), E, F can be made to vanish using some coordinate transformations, so the tetrad giving the structure of the space can be written in the following form:

$$\begin{aligned} \underset{0}{\lambda}^{0} &= A \\ \underset{0}{\lambda}^{\alpha} &= Dx^{\alpha} \\ \underset{a}{\lambda}^{\alpha} &= \delta^{\alpha}_{a}B + \varepsilon_{\alpha a\beta}x^{\beta}S \end{aligned} \tag{2}$$

As stated by Robertson (1932), improper rotation can be admitted if and only if

$$S = 0 \tag{3}$$

This implies a definite physical meaning according to the present theory (I), and will be discussed later. For the present work, we are going to study the case (3) for which (2), in spherical polar coordinates, has the following form:

$$\underset{i}{\lambda}^{\mu} \equiv \begin{array}{cc} {}_{i}\backslash^{\mu} & \begin{array}{cccc} 0 & 1 & 2 & 3 \end{array} \\ \begin{array}{c} 0 \\ 1 \\ 2 \\ 3 \end{array} & \begin{vmatrix} A & Dr & 0 & 0 \\ 0 & B\sin\theta\cos\phi & \dfrac{B}{r}\cos\theta\cos\phi & -\dfrac{B}{r}\dfrac{\sin\phi}{\sin\theta} \\ 0 & B\sin\theta\sin\phi & \dfrac{B}{r}\cos\theta\sin\phi & \dfrac{B}{r}\dfrac{\cos\phi}{\sin\theta} \\ 0 & B\cos\theta & -\dfrac{B}{r}\sin\theta & 0 \end{vmatrix} \end{array} \tag{4}$$

3. THE FIELD EQUATIONS

The field equations to be solved (I) are of the form

$$E_{\mu\nu} = 0 \tag{5}$$

where $E_{\mu\nu}$ is a second-order nonsymmetric tensor given by

$$\begin{aligned} E_{\mu\nu} \overset{\text{def}}{=} {} & g_{\mu\nu}L - 2L_{\mu\nu} - 2g_{\mu\nu}C^{\sigma}|_{\sigma} - 2C_{\mu}C_{\nu} \\ & -2g_{\mu\sigma}C^{\varepsilon}\Lambda^{\sigma}_{\varepsilon\nu} + 2C_{\nu\underset{+}{|}\mu} - 2g^{\sigma\varepsilon}\Lambda_{\mu\nu\varepsilon\underset{+++}{|}\sigma} \end{aligned} \tag{6}$$

and

$$\begin{aligned} L_{\mu\nu} &\overset{\text{def}}{=} \Lambda^{\sigma}_{\varepsilon\mu}\Lambda^{\varepsilon}_{\sigma\nu} - C_{\mu}C_{\nu} \\ L &\overset{\text{def}}{=} g^{\mu\nu}L_{\mu\nu} \\ C_{\mu} &\overset{\text{def}}{=} \Lambda^{\varepsilon}_{\mu\varepsilon} \\ \Lambda^{\varepsilon}_{\mu\nu} &\overset{\text{def}}{=} \Gamma^{\varepsilon}_{\mu\nu} - \Gamma^{\varepsilon}_{\nu\mu} \\ \Lambda_{\varepsilon\mu\nu} &\overset{\text{def}}{=} g_{\varepsilon\sigma}\Lambda^{\sigma}_{\mu\nu} \end{aligned} \tag{7}$$

The vertical bar denotes absolute differentiation using the nonsymmetric connection $\Gamma^{\sigma}_{\mu\nu}(=\underset{i}{\lambda}^{\sigma}\underset{i}{\lambda}_{\mu,\nu})$. The $(+)$ and the $(-)$ signs are used in the usual manner to distinguish between the two types of absolute derivatives. $g_{\mu\nu}(=\underset{i}{\lambda}_{\mu}\,\underset{i}{\lambda}_{\nu})$ is a symmetric tensor.

4. TYPE ANALYSIS AND STRENGTH OF THE FIELDS

Owing to the lengthy calculations of the model (4), especially when calculating the tensors of this section, the author has used an algebraic manipulation program to calculate these tensors and to check other calculations. The author wrote the program in REDUCE2 and it was run in MTS (Michigan Terminal System) at NUMAC (Newcastle).

Before solving the field equations, it is useful to carry out the type analysis for the space (4) [see (II), Table I]. This analysis gives an idea of the sort and the strength of fields that a space of given structure can represent. The tensors which are responsible of the type are found to have the following properties for the space (4)

$$\begin{aligned} R^{\mu}_{\nu\sigma\varepsilon} &\neq 0, & T_{\mu\nu} &= 0 \\ F_{\mu\nu} &\neq 0, & Z_{\mu\nu} &= 0 \\ & & \Lambda &= 0 \end{aligned} \tag{8}$$

where the tensors in (8) have been defined in the previously mentioned papers [cf. (II)].

If we use Table I in (II) we find that the tensors in (8) match the type FIGI. This means that the space of structure (4) can represent, at most (without any further condition) a generalized field which is not strong outside a spherically symmetric charged body. Of course one can find solutions corresponding to tetrads of less generality, using (4).

5. SOLUTIONS OF THE FIELD EQUATIONS

If we use (4) to evaluate the tensors (7) and substitute into (6), then the field equations (5) will give rise to the following set of differential equations ($A' \equiv dA/dr$, $A'' \equiv d^2A/dr^2, \ldots$)

$$\frac{B^2+D^2r^2}{A^2}\left[b(r)+\frac{D^2r^2}{B^2}\,l(r)\right]=0 \tag{9}$$

$$\frac{Dr}{A}\left[b(r)+\frac{D^2r^2}{B^2}\,l(r)\right]=0 \tag{10}$$

$$-\frac{B'^2}{B^2}+\frac{2}{r}\left(\frac{B'}{B}+\frac{A'}{A}\right)-2\frac{A'B'}{AB}-l(r)\frac{D^2r^2}{B^2}=0 \tag{11}$$

$$\begin{aligned}\frac{D^2r^2}{B^2}\Bigg[-l(r)+\frac{A''}{A}-\frac{D''}{D}+\frac{4}{r}\left(\frac{A'}{A}-\frac{D'}{D}\right)\\ -2\frac{A'^2}{A^2}-\frac{D'^2}{D^2}+3\left(\frac{A'D'}{AD}-\frac{A'B'}{AB}+\frac{B'D'}{BD}\right)\Bigg]\\ +\frac{A''}{A}+\frac{B''}{B}-2\frac{A'^2}{A^2}-\frac{B'^2}{B^2}+\frac{1}{r}\left(\frac{A'}{A}+\frac{B'}{B}\right)=0\end{aligned} \tag{12}$$

where

$$\begin{aligned}b(r)&=3\frac{B'^2}{B^2}-\frac{4}{r}\frac{B'}{B}-2\frac{B''}{B}\\ l(r)&=5\frac{B'^2}{B^2}-\frac{8}{r}\frac{B'}{B}-2\frac{B''}{B}+\frac{2}{r}\frac{D'}{D}-2\frac{B'D'}{BD}+\frac{3}{r^2}\end{aligned} \tag{13}$$

We are interested only in solutions which can give some physical meaning. So, we are going to look for such solutions in the following manner. Consider the generalized electromagnetic potential C_μ [cf. (I)], for (4). This vector has the following nonvanishing components:

$$C_0=\frac{Dr}{A}\left(\frac{B'}{B}-\frac{D'}{D}-\frac{3}{r}\right) \tag{14}$$

$$C_1=\frac{A'}{A}+2\frac{B'}{B} \tag{15}$$

The vanishing of C_0 only will give rise to the vanishing of the electromagnetic field, since C_1 is a function of x^1 only and consequently it could not give rise to any skew tensor. This can be achieved by taking $D=0$. The vanishing of C_1 alone will not affect the existence of the electromagnetic field. This can be done by taking $A'/A=-2(B'/B)$. We are also interested in the solution where both C_0, C_1 vanish.

Although those are not the only solutions which can be obtained, yet, for reasons which will appear in due course, and for the time being, we are not going to study other solutions.

5.1. The solution $C_0=0$

Since D is one of the unknown functions, the condition $D=0$ will affect the type of the space. In the present case ($D=0$), the tensors (8) are now found to be such that

$$
\begin{aligned}
R^{\mu}_{\nu\sigma\varepsilon} &\neq 0\\
T_{\mu\nu} &= 0\\
\Lambda &= 0\\
F_{\mu\nu} &= 0\\
Z_{\mu\nu} &= 0
\end{aligned}
\tag{16}
$$

So, unless other conditions are found from the solution, (16) will match the type FOGI. This represents only a gravitational field, which is not strong, outside a spherical symmetric body [see (II), Table (I)].

The equations to be solved [(9)-(12)] are now reduced to ($D=0$)

$$
\begin{aligned}
2\left(\frac{B'}{B}\right)' - \frac{B'^2}{B^2} + \frac{4}{r}\frac{B'}{B} &= 0\\
\frac{2}{r}\left(\frac{A'}{A}+\frac{B'}{B}\right) - \frac{B'}{B}\left(2\frac{A'}{A}+\frac{B'}{B}\right) &= 0\\
\left(\frac{A'}{A}\right)' - \frac{A'^2}{A^2} + \left(\frac{B'}{B}\right)' + \frac{1}{r}\left(\frac{A'}{A}+\frac{B'}{B}\right) &= 0
\end{aligned}
\tag{17}
$$

The solution which satisfies (17) is found to have the form

$$
\begin{aligned}
B &= \frac{1}{\alpha_1(1+m/2r)^2}\\
A &= \alpha_2 \frac{1+m/2r}{1-m/2r}
\end{aligned}
$$

where α_1, α_2, m are constants. Since the model is spherically symmetric, we need $\underset{i}{\lambda}^{\mu} \to \delta^{\mu}_{i}$ as $r \to \infty$. This can be achieved if $\alpha_1 = \alpha_2 = 1$. Then the solution can be written in the following form:

$$
\begin{aligned}
A &= \frac{1+m/2r}{1-m/2r}\\
B &= \frac{1}{(1+m/2r)^2}\\
D &= 0
\end{aligned}
\tag{18}
$$

This solution has been obtained before (III) using a simpler model and is found to represent the gravitational field outside a spherical symmetric body as expected from the type analysis.

5.2. The Solution $C_0 = 0$ and $C_1 = 0$

In this case, the differential equations to be satisfied are (17) together with the condition ($C_1 = 0$)

$$\frac{A'}{A} = -2\frac{B'}{B} \tag{19}$$

As shown before the solution (18) has been found to satisfy the set (17). It can easily be shown that by using (18), (19) cannot be satisfied unless $m = 0$. So the solution will be reduced in this case to

$$\begin{aligned} A &= 1 \\ B &= 1 \\ D &= 0 \end{aligned} \tag{20}$$

Using these values we found that the tensors given by (16) all vanish. This matches the type FOGO which is the flat space-time of special relativity.

5.3. The Solution $C_1 = 0$

This condition ($C_1 = 0$) will not affect the type, i.e., the tetrad (4) will preserve its general type FIGI. The field equations to be solved are now (9) [≡(10)], (11) and (12) together with the condition given by (19). Eliminating $l(r)$ between (9), (11) we get

$$\left(\frac{B'}{B}\right)' + \frac{A'B'}{AB} + \frac{1}{r}\left(\frac{B'}{B} - \frac{A'}{A}\right) = 0 \tag{21}$$

Using (19) to eliminate A'/A in the last equation, we get by integration

$$B^2 = \frac{1}{(\alpha_3 + \beta/r^2)} \tag{22}$$

Substituting from (22) into (21) we get

$$A = \alpha_4(\alpha_3 + \beta/r^2) \tag{23}$$

where α_3, α_4, β are constants.

Using (22) into (9) we get for D a first-order differential equation which can be written in the form

$$2DD' + \frac{3\alpha_3 r^2 + 4\beta}{\alpha_3 r^3 + \beta r} D^2 = \frac{\beta^2 - 2\beta\alpha_3 r^2}{\alpha_3 r^3(\alpha_3 r^2 + \beta)^2}$$

The solution of this equation is found to be

$$D^2=\frac{\alpha(\alpha_3 r^2+\beta)^{3/2}+\gamma(2\alpha_3 r^2+\beta)}{r^4(\alpha_3 r^2+\beta)} \tag{24}$$

where γ is constant. The solution given by (22), (23), and (24) is found to satisfy (11) if

$$\gamma=\frac{\beta}{\alpha_3^2}$$

and since the present tetrad is spherically symmetric we require, as stated before, $\underset{i}{\lambda}^{\mu}\to\delta_i^{\mu}$ as $r\to\infty$. So we should take $\alpha_3=\alpha_4=1$, consequently $\gamma=\beta$. The solution now can be written in the form

$$\begin{aligned} A^2&=\left(1+\frac{\beta}{r^2}\right)^2\\ B^2&=\frac{1}{1+\beta/r^2}\\ D^2&=\frac{\alpha(r^2+\beta)^{3/2}+\beta(2r^2+\beta)}{r^4(r^2+\beta)} \end{aligned} \tag{25}$$

which satisfies (12) without any further condition. This solution is found to represent (as expected from the space type) a generalized field outside a spherically symmetric charged body. This will be discussed in the following section.

6. PHYSICAL INTERPRETATION AND COMPARISON WITH OTHER THEORIES

As has been shown in Sections 4 and 5 the type analysis can be used to gain some physical information about the problem concerned even before solving the field equations. To summarize, three solutions have been obtained: the solution (18), giving rise to FOGI, represents a gravitational field outside a spherical body; the solution (20), giving rise to FOGO, represents the empty space-time of special relativity; while the solution (25), with the type FIGI, represents a generalized field outside a spherical charged body.

6.1. The Metric Analogy

Now to support the previous physical information and to gain more, we are going to write the metric of the real domain of Riemannian space associated with (4) for each of the solutions obtained. This metric can be defined (III) as

$$ds^2 = \overset{*}{g}_{\mu\nu}\, dx^\mu\, dx^\nu$$

where

$$\overset{*}{g}_{\mu\nu} = \sum_i e_i \underset{i}{\lambda}_\mu \underset{i}{\lambda}_\nu \tag{26}$$

and

$$e_i = (1, -1, -1, -1)$$

(1) Using (26), (4) we can write for the solution (18) the metric

$$ds^2 = \frac{(1-m/2r)^2}{(1+m/2r)^2}\, dt^2 - (1+m/2r)^4 (dr^2 + d\sigma^2) \tag{27}$$

where $d\sigma^2 = r^2\, d\theta^2 + r^2 \sin^2\theta\, d\phi^2$, which is identical with Schwarzschild exterior metric in its isotropic form. This solution has been obtained before (III) using a simple model, so we are not going to discuss it.

(2) For the solution (20) we get the metric

$$ds^2 = dt^2 - (dr^2 + d\sigma^2) \tag{28}$$

which is the empty space-time of special relativity. This is exactly what has been given by the type analysis.

(3) For the solution (25) we get the metric [$A = 1/B^2$ as a consequence of (25)]

$$ds^2 = B^2(B^2 - D^2 r^2)\, dt^2 + 2Dr\, dr\, dt - B^{-2}(dr^2 + d\sigma^2)$$

Using a coordinate transformation of the form

$$T = t + F(r), \qquad F(r) = \int \frac{Dr}{B^2(\beta^2 - D^2 r^2)}\, dr$$

$$R = \frac{r}{B}$$

The last metric can be written into the form

$$ds^2 = \gamma(R)\, dt^2 - \frac{dR^2}{\gamma(R)} - R^2\, d\theta^2 - R^2 \sin^2\theta\, d\phi^2 \tag{29}$$

where

$$\gamma(R) = 1 - \frac{\alpha}{R} + \frac{4\beta}{R^2} + \frac{2\beta^2}{R^4} \tag{30}$$

To give a physical meaning for the constants α, β of the solution (25), let $\beta = 0$. Then (30) will take the form

$$\gamma(R) = 1 - \frac{\alpha}{R}$$

and (29) will be of the form of Schwarzschild exterior metric and we can take $\alpha = 2m$ where m is the mass of the object in relativistic units ($c = G = 1$). We can surmise that β is proportional to the charge of the body.

This suggestion is supported if we consider the case when R is large enough so that we can neglect quantities of the order $(1/R^4)$. In this case (30) will take the form

$$\gamma(R) \simeq 1 - \frac{\alpha}{R} - \frac{4\beta}{R^2} \tag{31}$$

which can be compared with the well-known Reissner–Nordström metric for a charged point mass, viz.,

$$ds^2 = \gamma_1(R)\, dt^2 - \frac{dR^2}{\gamma_1(R)} - R^2\, d\theta^2 - R^2 \sin^2\theta\, d\phi^2$$

with

$$\gamma_1(R) = 1 - \frac{2m}{R} + \frac{Ke^2}{2R^2} \tag{32}$$

From (31), (32) we get

$$\alpha = 2m, \qquad \beta = -\frac{Ke^2}{8} \tag{33}$$

So, for (29), $\gamma(R)$ will take the form

$$\gamma(R) = 1 - \frac{2m}{R} + \frac{Ke^2}{2R^2} + \frac{K^2e^4}{32R^4} \tag{34}$$

Taking $m = GM/c^2$, $K = 8\pi G/c^4$ we get (M is the mass in grams)

$$\gamma(R) = 1 - \frac{2MG}{c^2R} + \frac{4\pi Ge^2}{c^4R^2} + \frac{2\pi^2G^2e^4}{c^8R^4} \tag{35}$$

6.2. Comparison with Other Field Theories

In comparing the present theory (I) with other field theories, in similar cases, the following results have been found:

(i) The same results, from the metric point of view, of *general relativity* has been obtained from the present theory in the case of spherical symmetry, i.e., the solution (18) with the metric (27).

(ii) As has been shown above, the metric (29) associated with the solution (25) is found to be similar to the Reissner–Nordström metric far from the object, so the present theory agrees with *Einstein-Maxwell theory* in the case of spherical symmetry with two main differences:

(a) Near the object, the present theory gives an extra term (over Einstein–Maxwell theory) in the gravitational potential, i.e., the last term in (34).

(b) All solutions obtained in the present work, are exterior solutions, i.e., with vanishing material-energy tensor [defined in (I)], as is clear from the general type FIGI of (4). This is not surprising since fields represented by (4) are weak (as clear from the type), and as has been shown (II), weak fields make no contribution to that tensor.

(iii) Einstein and Mayer (1930) have used a tetrad of the same type as (4) to test the so called *Einstein–Cartan* theory of absolute parallelism. However, Blackwell (1932) has shown that the solution obtained does not tend to the Schwarzschild exterior solution after eliminating the electric charge. The present solution (25) satisfies this requirement as has been shown in Section 6.1.

(iv) Mikhail (1964) has used a tetrad identical with (4) to solve the field equations of his *unified field theory*. The only solution which has been obtained represents pure gravity. From the present theory, solutions corresponding to both gravity and electromagnetism have been obtained.

(v) Tonnelat (1966) has discussed a spherically symmetric solution (of Papapetrou type) of the *Einstein nonsymmetric theory*. She noticed that (p. 99) as the distance from the object becomes large, the solution does not agree with Reisner–Nordström metric. She expected that the results of any unified field theory can be reduced to that of general relativity combined with Maxwell's theory at large distances. This is exactly what has been shown for the present theory in Section 6.1.

7. DISCUSSION AND CONCLUSION

The space used in the present application (4) has spherical symmetry, and as shown before with vanishing material-energy tensor. Physical problems with spherical symmetry are: the gravitational field outside a spherically symmetric body; a generalized field outside a spherically symmetric charged body; and the trivial case of empty space-time. The three solutions obtained are found to represent the previously mentioned problems. For this reason we are not going to examine possible other solutions. We expect other solutions of (9)–(12), if any, to be either identical to the solutions already obtained or to deviate sharply from classical known results. In this last case we cannot use classical results for comparison. We can compare only with the solutions obtained in the present work. So we consider the present work as a necessary step for the future work.

All solutions obtained in the present work, correspond to (3) $S(r)=0$. As stated in Section 2 this corresponds to improper rotation. It has been found that if $S(r)\neq 0$, the type of the space will be changed, giving rise to strong fields within a material distribution.

Tonnelat (1966), when discussing the Einstein nonsymmetric theory, has expected that the unified laws contain additional terms which represent interaction of the gravitational and electromagnetic fields. These terms, as she stated, can give rise to a creation of a magnetic field by a purely material distribution (Blackett effect). However, as shown in Section 6, the solution (25) tends to that of Schwarzschild when $\beta=0$ (or $e=0$). This is correct from the point of view of relativity theory (the metric only). But we have found that $\beta=0$ corresponds to $F_{\mu\nu}\neq 0$. This is not surprising from the point of view of a unified field. For, as the electromagnetic field has its contribution to the gravitational potential [extra terms due to β in (34)], we expect the gravitational field to contribute to the electromagnetic potential as well (and of course to $F_{\mu\nu}$). This type of interaction has been predicted before (II) in studying weak fields. This interaction may give rise to an effect similar to Blackett effect when the model (with $e=0$) is rotated.

In summary the three solutions obtained in the present work are in agreement with classical known results except for an extra term in the case of a generalized field. Two methods can be used to get the physical meaning: the first is to use the type analysis (II) to get some physical information about the geometrical space used. The second is to write the metric of the real domain of the associated Riemannian space, in order to compare with classical known results. The second method cannot be used unless the solution of the field equations is already obtained.

There remains for further investigation: the possibility of other solutions of (9)–(12); the solution in the case of strong fields ($S\neq 0$); and the possibility of prediction of a phenomenon like the Blackett effect.

ACKNOWLEDGMENTS

The author wishes to thank the authorities of King's college for giving him necessary funds to use computer facilities. Moreover he would like to thank Professor W. H. McCrea F.R.S., and Professor C. W. Kilminster for many discussions.

REFERENCES

Blackwell, A. (1932). *Proceedings of the Royal Society of Edinburgh*, **52**, 327.

Einstein, A., and Mayer, W. (1930). *Sitzungsberichte der Preussischen Akadademie der Wissenschaften*, **1**, 110.

Mikhail, F. I. (1964). *II Nuovo Cimento X*, **32**, 886.

Mikhail, F. I., and Wanas, M. I. (1977). *Proceedings of the Royal Society of London Series A*, **356**, 471.

Mikhail, F. I., and Wanas, M. I. (1981). *International Journal of Theoretical Physics*, **20**, 9, 671.

Robertson, H. P. (1932). *Annals of Mathematics Princeton (2)*, **33**, 496.

Tonnelat, M. A. (1966). *Einstein's Unified Field Theory*. Gordon and Breach, New York.

Wanas, M. I. (1981). *Il Nuovo Cimento B*, **66**, 2, 145.